Ch 2 Assignment -

2.1 # 16, 36, 46, 52, 72, 74 ; 2.2 # 2, 24, 34, 36, 60ac

2.3 #16, 18, 20 ; 2.4 # 2, 10, 26, 38, 58 ; 2.5 # 2, 4, 10, 16, 24

2.6 # 4, 18, 44, 46

$\boxed{2.1}$ 16) $f(x) = 2x + 4$ at $(1, 6)$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{[2(x+\Delta x)+4] - [2x+4]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x + 2\Delta x + 4 - 2x - 4}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 2$$

$$\boxed{= 2}$$

Note: At $(1, 6)$, the slope is also 2. In this setting, the slope is constant for $f(x)$.

36) $f(t) = t^3 + t^2$

$$\lim_{\Delta t \to 0} \frac{[(t+\Delta t)^3 + (t+\Delta t)^2] - [t^3 + t^2]}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\cancel{t^3} + 3t^2\cancel{\Delta t} + 3t(\Delta t)^{\cancel{2}} + (\Delta t)^{\cancel{3}\,2} + \cancel{t^2} + 2t\cancel{\Delta t} + (\Delta t)^{\cancel{2}} - \cancel{t^3} - \cancel{t^2}}{\cancel{\Delta t}}$$

$$= \lim_{\Delta t \to 0} \left(3t^2 + 3t\Delta t + (\Delta t)^2 + 2t + \Delta t\right)$$

$$\boxed{= 3t^2 + 2t}$$

46) $f(x) = \frac{1}{x-1}$ at $(2,1)$

$$\lim_{\Delta x \to 0}\left(\frac{1}{(x+\Delta x-1)} - \frac{1}{x-1}\right) \Big/ \Delta x$$

$$= \lim_{\Delta x \to 0} \frac{(x-1)-(x+\Delta x-1)}{(x+\Delta x-1)(x-1)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}}{(x-1)(x+\Delta x-1)\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{(x-1)(x+\Delta x-1)}$$

$$= -\frac{1}{(x-1)^2}$$

$$f'(2) = -\frac{1}{(2-1)^2} = -1$$

$$(y-y_0) = m(x-x_0)$$

$$(y-1) = -1(x-2)$$

$$y = -x+2+1$$

$$\boxed{y = -x+3}$$

52) There are cusps at $x = \pm 3$
The function is differentiable on $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

72) False. Consider problem 52 where there are cusps at ± 3.

74) True

$\boxed{2.2}$ 2) $y = x^{3/2}$

$$y' = \frac{3}{2}x^{1/2}$$

At $(1,1)$, $y' = \frac{3}{2}$

24)

Function	Rewrite	Differentiate	Simplify
$y = \frac{2}{3x^2}$	$y = \frac{2}{3}x^{-2}$	$y' = \frac{2}{3}(-2)x^{-3}$	$y' = -\frac{4}{3x^3}$

34) $f(x) = 3(5-x)^2$ at $(5,0)$

$= 3(25 - 10x + x^2)$

$= 125 - 30x + 3x^2$

$f'(x) = -30 + 6x$

$f'(5) = -30 + 6(5) \boxed{= 0}$

36) $f(x) = x^2 - 3x - 3x^{-2} + 5x^{-3}$

$f'(x) = 2x - 3 + 6x^{-3} - 15x^{-4}$

60) a) $L = -38.8333t^4 + 302.944t^3 - 538.50t^2 + 209.7t + 12652$

$L' = -38.8333(4t^3) + 302.944(3t^2) - 538.50(2t) + 209.7$

$= -155.3332t^3 + 908.832t^2 - 1077.0t + 209.7$

For 2002, $t=2$:

$L' = -155.3332(8) + 908.832(4) - 1077(2) + 209.7$

$\boxed{= 448.3624}$

For 2004, $t=4$:

$L' = -155.3332(64) + 908.832(16) - 1077(4) + 209.7$

$\boxed{= 964.1916}$

c) L is in millions of cubic feet per year, t is year

So the units for $\frac{dL}{dt}$ are $\frac{\text{millions of } ft^3/\text{year}}{\text{year}}$

or $\frac{\text{millions of } ft^3}{\text{year}^2}$

$\boxed{2.3}$ 16) $C = 1{,}008{,}000/Q + 6.3Q$

Avg. rate of change on $[350, 351]$ is

$\frac{f(351)-f(350)}{351-350} = \left(1008000/351 + 6.3(351)\right) - \left(1{,}008{,}000/350 + 6.3(350)\right)$

$= 5083.095 - 5085$

$\boxed{= -1.905}$

Instantaneous rate of change is C' at $Q = 350$:

$$C = 1{,}008{,}000Q^{-1} + 6.3Q$$

$$C' = -1{,}008{,}000Q^{-2} + 6.3$$

At $Q = 350$, $C' = -1{,}008{,}000(350)^{-2} + 6.3$

$$\boxed{= -1.9286}$$

The rates of change are similar

18) $E = \frac{1}{27}(9t + 3t^2 - t^3)$, $0 \le t \le 4.5$

$$= \frac{1}{3}t + \frac{1}{9}t^2 - \frac{1}{27}t^3$$

$$E' = \frac{1}{3} + \frac{2}{9}t - \frac{1}{9}t^2$$ <for instantaneous rate of change>

a) $[0, 1]$

$$\frac{f(b) - f(a)}{b - a} = \frac{\left[\frac{1}{3}(1) + \frac{1}{9}(1)^2 - \frac{1}{27}(1)^3\right] - \left[\frac{1}{3}(0) + \frac{1}{9}(0)^2 - \frac{1}{27}(0)^3\right]}{1 - 0}$$

$$= \frac{1}{3} + \frac{1}{9} - \frac{1}{27} = \frac{9}{27} + \frac{3}{27} - \frac{1}{27}$$

$$= \frac{11}{27} \approx 0.4074$$

$E'(0) = \frac{1}{3} \approx 0.3333$ $\qquad$ $E'(1) = \frac{1}{3} + \frac{2}{9} - \frac{1}{9} = \frac{4}{9} \approx 0.4444$

b) $[1, 2]$

$$\frac{f(2) - f(1)}{2 - 1} = \left[\frac{1}{3}(2) + \frac{1}{9}(2)^2 - \frac{1}{27}(2)^3\right] - \frac{11}{27}$$

$$= \frac{2}{3} + \frac{4}{9} - \frac{8}{27} - \frac{11}{27}$$

$$= \frac{18}{27} + \frac{12}{27} - \frac{8}{27} - \frac{11}{27} = \frac{22}{27} - \frac{11}{27} = \frac{11}{27} \approx 0.4074$$

$E'(1) \approx 0.4444$ $\qquad$ $E'(2) \approx 0.3333$

c) $[2, 3]$

$$\frac{f(3) - f(2)}{3 - 2} = \frac{1}{3}(3) + \frac{1}{9}(3)^2 - \frac{1}{27}(3)^3 - \frac{22}{27}$$

$$= \frac{27}{27} - \frac{22}{27} = \frac{5}{27} \approx 0.1852$$

$E'(2) \approx 0.3333 \qquad E'(3) = 0$

d) $\frac{f(4)-f(3)}{4-3} = \frac{1}{3}(4) + \frac{1}{9}(4)^2 - \frac{1}{27}(4)^3 - \frac{27}{27}$

$= \frac{20}{27} - \frac{27}{27} = \frac{-7}{27} \approx 0.2593$

$E'(3) = 0 \qquad E'(4) \approx -0.5556$

20) $s(t) = -16t^2 + 555$

a) $\frac{s(3)-s(2)}{3-2} = (-16(3)^2 + 555) - (-16(2)^2 + 555)$

$= 16(4-9) = 16(5)$

$= 80$ ft/sec

b) $s'(t) = -32t$

$s'(2) = -32(2) = -64$ ft/sec

$s'(3) = -32(3) = -96$ ft/sec

c) Hits ground at $s(t) = 0$

$0 = -16t^2 + 555$

$16t^2 = 555$

$t^2 = 34.6875$

$t = \pm 5.8896$

After 5.8896 seconds.

d) $v(t) = s'(t) = -32t$

$v(5.8896) = -32(5.8896) = -188.4675$ ft/sec.

$\boxed{24}$ 2) $g(x) = \underbrace{(x-4)}_{f}\underbrace{(x+2)}_{h}$. use product rule

$g'(x) = f(x)h'(x) + h(x)f'(x)$

$= (x-4)(1) + (x+2)(1)$

$= x - 4 + x + 2$

$= 2x - 2$

At $(4,0)$, $g'(4) = 8 - 2$

$= 6$

4) $f(x) = (x^2+1)(2x+5)$ use product rule

$$f'(x) = (x^2+1)(2) + (2x+5)(2x)$$
$$= 2x^2+2+4x^2+10x$$
$$= 6x^2+10x+2$$

Oops- looked at the wrong section- here's a bonus example :)

At $(-1, 6)$

$$f'(-1) = 6-10+2 = -2$$

10) $h(x) = \dfrac{x^2}{x+3}$ at $(-1, \frac{1}{2})$

Use quotient rule:

$$h'(x) = \frac{(x+3)(2x) - x^2(1)}{(x+3)^2} = \frac{2x^2+6x-x^2}{(x+3)^2} = \frac{x^2+6x}{(x+3)^2}$$

At $(-1, \frac{1}{2})$,

$$h'(-1) = \frac{(-1)^2+6(-1)}{(-1+3)^2} = \frac{1-6}{2^2} = \frac{-5}{4}$$

26) $h(t) = (t^5-1)(4t^2-7t-3)$

Use product rule:

$$h'(t) = (t^5-1)(8t-7) + (4t^2-7t-3)(5t^4)$$
$$= 8t^6-7t^5-8t+7+20t^6-35t^5-15t^4$$
$$= 28t^6-42t^5-15t^4-8t+7$$

38) $f(x) = \dfrac{x+1}{\sqrt{x}}$

Use quotient rule: $f(x) = \dfrac{x+1}{x^{1/2}}$

$$f'(x) = \frac{x^{1/2}(1) - (x+1)\left(\frac{1}{2}x^{-1/2}\right)}{(x^{1/2})^2} = \frac{x^{1/2} - \frac{1}{2}x^{1/2} + \frac{1}{2}x^{-1/2}}{x}$$

$$= x^{-1/2} - \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$$

$$= \frac{1}{\sqrt{x}} - \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt[3]{x}}$$

58) $P = (t-12)(3t^2-20t) + 250$

$P' = (t-12)(6t-20) + (3t^2-20t)(1)$

At $t=2$,

$P' = (2-12)(12-20) + (12-40)$

$= -10(-8) + (-28)$

$= 80 - 28$

$=$ 52 million microbes/hour

$\boxed{2.5}$ 2) $y = (x^2-2x+3)^3$

$u = x^2 - 2x + 3$ $\quad f(u) = u^3$

4) $y = (x^2+1)^{4/3}$

$u = x^2+1$ $\quad f(u) = u^{4/3}$

10) $y = u^3$, $\quad u = 3x^2 - 2$

$\frac{dy}{du} = 3u^2$ $\quad \frac{du}{dx} = 6x$ $\quad \frac{dy}{dx} = 3(6x)^2 = 3 \times 36x^2 = 108x^2$

16) $f(x) = \frac{2x}{1-x^3}$ $\quad$ d) Quotient rule

24) $y = (2x^3+1)^2$

$y' = 2(2x^3+1)(6x^2)$

$= 12x^2(2x^3+1)$

$\boxed{2.6}$ 4) $f(x) = 3x^2 + 4x$

$f'(x) = 6x + 4$

$f''(x) = 6$

18) $f(x) = x^4 - 2x^3$

$f'(x) = 4x^3 - 6x^2$

$f''(x) = 12x^2 - 12x$

$f'''(x) = 24x - 12$

44) $-16t^2 + 1250$ position

$v(t) = -32t$ velocity

$a(t) = -32$ acceleration

b) $0 = -16t^2 + 1250$

$16t^2 = 1250$

$t^2 = 78.125$

$t = \pm 8.8388 \rightarrow$ use $t = 8.8388$ sec (time must be positive)

c) $v(8.8388) = -32(8.8388) = -282.8427$ feet/sec.

46) $S = -8.25t^2 + 66t$

$v(t) = -16.5t + 66$

$a(t) = -16.5$

"given values of t" appear to be missing. I just picked some. My goal was to see where the car stops!

t	0	1	2	3	4
position	0	57.75	99	123.75	132
velocity	66	49.9	33	16.5	0
acceleration	-16.5	-16.5	-16.5	-16.5	

Note: If we're applying brakes to stop, the range of $v(t)$ is $0 \leq v(t) \leq 66$.

The car stops after 132 feet.