1.3 Lines in the Plane and Slope

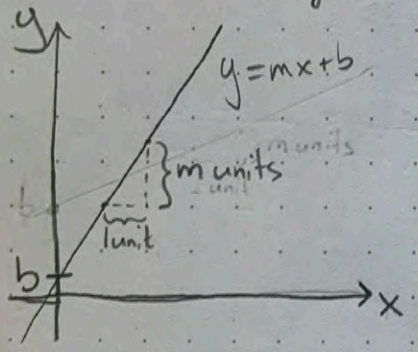
learning outcomes:
- Use the slope-intercept form of a linear equation to sketch graphs.
- Find slopes of lines passing through two points.
- Use the point-slope form to write equations of lines.
- Find equations of parallel and perpendicular lines.
- Use linear equations to model and solve real-life problems.

> Using Slope

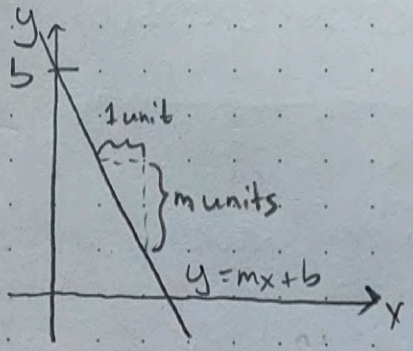
We can relate two variables using the linear equation
$y = mx + b$ (straight line ≡ linear)

The y-intercept is $y = m(0) + b = b$ $(0, b)$

The slope (steepness) of the line is m. Slope is the number of units a line rises (falls) vertically for each unit of horizontal change.



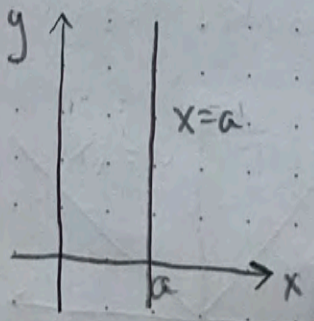
$m > 0$
Positive slope



$m < 0$
Negative slope

The Slope-Intercept form of a line:
The graph of the equation
$y = mx + b$
is a line with slope m and y-intercept $(0, b)$.

A vertical line is of the form
$x = a$
Its slope is undefined.
"divide by zero"



Ex Sketch the graph of $x+y=2$

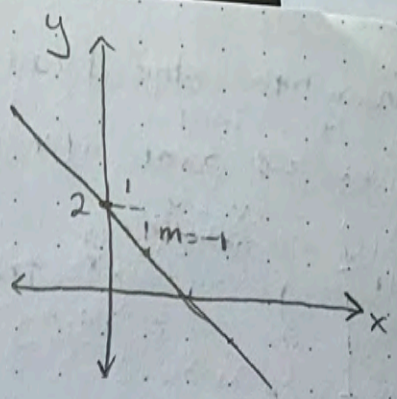
Soln 1) Convert to slope-intercept form:

$$x+y=2$$
$$y=-x+2$$
$$=(-1)x+2$$

2) Graph

Slope $=-1$

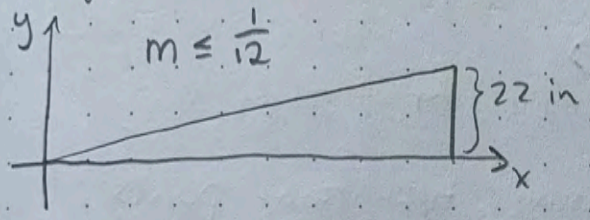
Y-intercept $=(0,2)$



Slope is often represented by a ratio or rate.

Ex Slope as ratio

The maximum recommended slope of a wheelchair ramp is $\frac{1}{12} \approx 0.083$. A business wants to build a ramp to reach a height of 22 inches. How much horizontal length is required?



Soln

$$\text{Slope} = \frac{\text{Vertical change}}{\text{horizontal change}} = \frac{1}{12}$$
$$\Rightarrow \frac{22\text{ in}}{H} = \frac{1}{12}$$
$$\Rightarrow 22\text{ in} = \frac{H}{12}$$
$$\Rightarrow 12 \times 22\text{ in} = H$$
$$\Rightarrow H = 264\text{ in or } 22\text{ feet}$$

The ramp must start <u>at least</u> 22 feet from the platform.

Ex Slope as rate (of change)

A killer whale calf's body length can be modeled as

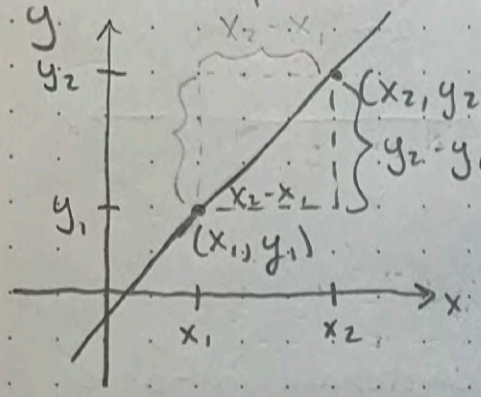
$$L = 10t + 260$$

where t = age in months.

- At $t=0$, the y-intercept represents the calf's body length at birth.
- The slope $m=10$ is the rate of change - a calf grows 10 centimeters per one month of age.

> Finding the slope of a line

Given two points, (x_1, y_1) and (x_2, y_2)



$$\frac{\text{Vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

We can denote change using Δ.

$\Delta x = x_2 - x_1$ <change in x>

Def The slope of a nonvertical line passing b/w two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: Either point may be defined as (x_1, y_1) or (x_2, y_2)

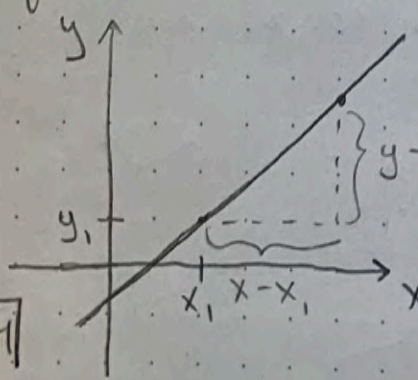
Ex Find the slope of the line passing through $(-2, 0)$ and $(3, 1)$

Soln Let $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (3, 1)$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{3 - (-2)} = \frac{1}{5}$$

> Writing Linear Equations

Given the slope and a single point on the line, we can find the equation for the line.



$$m = \frac{y - y_1}{x - x_1}$$

This gives the point-slope form of the eqn of a line:

Def The equation of a line with slope m passing thru point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

Ex Find the equation of the line with slope 3 that passes through the point $(1, -2)$.

Soln

$y - y_1 = m(x - x_1)$ $\quad (x_1, y_1) = (1, -2)$

$y - (-2) = 3(x - 1)$

$y + 2 = 3(x - 1)$ $\quad$ point-slope form

$y + 2 = 3x - 3$

$y = 3x - 5$ $\quad$ slope-intercept form

Note Since

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

we can rewrite the point-slope form as

$$y - y_1 = \frac{\Delta y}{\Delta x}(x - x_1) \quad \text{<two-point form>}$$

to find the line that passes between two points.

Finally, every line (incl vertical) can be written in the form

$$Ax + By + C = 0 \quad \text{<general form>}$$

> Parallel and Perpendicular Lines

Def Two distinct nonvertical lines are parallel iff their slopes are equal, $m_1 = m_2$.

Def Two distinct nonvertical lines are perpendicular iff their slopes are negative reciprocals of each other:

$$m_1 = -\frac{1}{m_2}$$

Ex Find the slope-intercept form of the equations of the lines that pass through $(2, -1)$ and are (a) parallel and (b) perpendicular to $2x - 3y = 5$.

Soln $2x - 3y = 5$

$-3y = -2x + 5$

$y = \frac{2}{3}x - \frac{5}{3}$

a) The parallel line has slope $m = \frac{2}{3}$

$y - y_1 = m(x - x_1)$

$y - (-1) = \frac{2}{3}(x - 2)$

$y + 1 = \frac{2}{3}x - \frac{4}{3}$

$y = \frac{2}{3}x - \frac{7}{3}$

b) The perpendicular line has slope $m = -\frac{3}{2}$

$y - (-1) = -\frac{3}{2}(x - 2)$

$y + 1 = -\frac{3}{2}x + 3$

$y = -\frac{3}{2}x + 2$

> Application: Linear Depreciation

A medical lab buys a $20,000 analytical balance scale with an estimated useful life of 8 years. Its salvage value is $4000. Write a linear equation to describe its book value each year.

Soln Let V = machine's value and t = year. Points of form (t, V). We have $(0, 20000)$ and $(8, 4000)$.

$$m = \frac{4000 - 20000}{8 - 0} = \frac{-16000}{8} = -2000$$

<$2000 decrease in value each year>

$V - 20000 = -2000(t - 0)$

$V = -2000t + 20,000 \quad , \quad 0 \le t \le 8$

So after 3 years, the scale's value is

$V = -2000(3) + 20,000 = \$14,000$