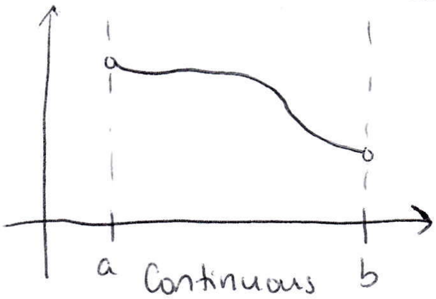
Continuity

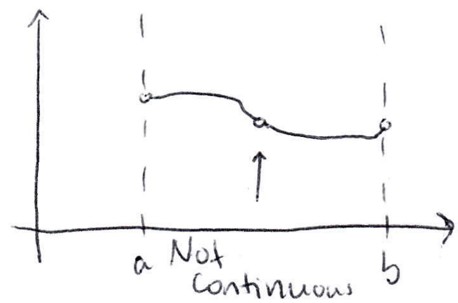
Def: Let c be a number in the interval (a,b) and let f be a function whose domain contains the interval (a,b). The function f is <u>continuous at c</u> if the following are (all) true:

1) $f(c)$ is defined
2) $\lim_{x \to c} f(x)$ exists
3) $\lim_{x \to c} f(x) = f(c)$.

<u>Note:</u> if direct substitution can be used to evaluate the limit of a function at c, the function is continuous at c.

If f is continuous on every point in the interval (a,b), then it is <u>continuous on the open interval</u> (a,b).





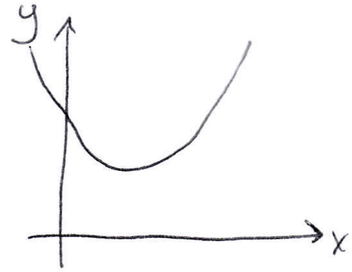
Continuity of polynomial and rational functions:

1) A polynomial function is continuous at every real number.
2) A rational function is continuous at every number in its domain.

Ex Consider $f(x) = x^2 - 2x + 3$.

This is continuous at every point!

continuous on $(-\infty, \infty)$

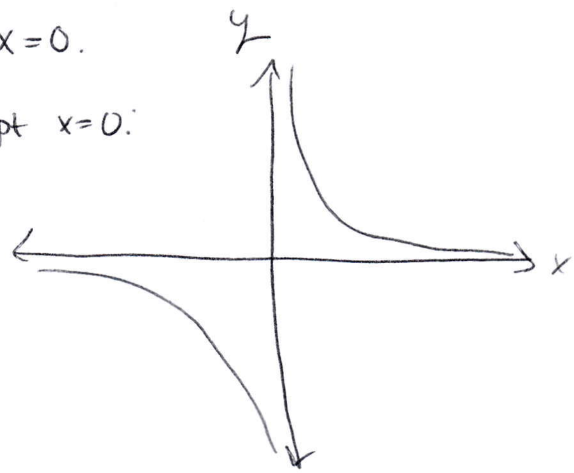


Ex Consider $f(x) = \frac{1}{x}$

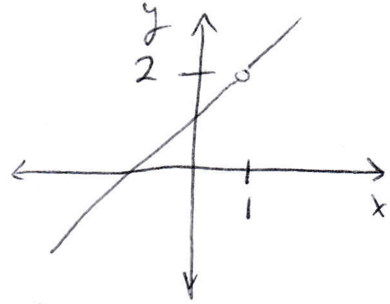
Its domain is all real numbers <u>except</u> $x = 0$.

This is continuous at every point except $x=0$.

Continuous on $(-\infty, 0)$ and $(0, \infty)$.



ex $f(x) = \frac{x^2-1}{x-1}$ has domain all reals except $x=1$



So it is continuous on $(-\infty, 1)$ and $(1, \infty)$

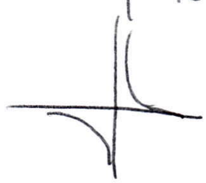
Def ~~A~~ For a function f which is not continuous at c, we say that it has a <u>discontinuity at c</u>.

Def A discontinuity at c is <u>removable</u> if f can be made continuous by (re)defining $f(c)$.

For $f(x) = \frac{x^2-1}{x-1}$, we could define $f(1) = 2$

Def A discontinuity at c is nonremovable if f cannot be made continuous by (re) defining $f(c)$.

For $f(x) = \frac{1}{x}$, there is no way to redefine $f(0)$ to make this continuous



<u>Continuity on a closed interval</u>

Def Let f be defined on a closed interval $[a,b]$. If f is continuous on the open interval (a,b) and

$$\lim_{x \to a^+} f(x) = f(a) \quad \text{and} \quad \lim_{x \to b^-} f(x) = f(b)$$

then f is <u>continuous on the closed interval</u> $[a,b]$.

Moreover, f is <u>continuous from the right</u> at a and <u>continuous from the left</u> at b.

Similar def's can be constructed for continuity on $(a, b]$, $[a, b)$ and on infinite intervals.

Ex Consider $f(x) = \sqrt{3-x}$. The domain is $(-\infty, 3]$.

$$\lim_{x \to 3^-} \sqrt{3-x} = 0$$
$$= f(3)$$

So $f(x)$ is continuous from the left at $x = 3$

and f is continuous on the interval $(-\infty, 3]$.

Ex Consider $g(x) = \begin{cases} 5-x & -1 \le x \le 2 \\ x^2-1 & 2 < x \le 3 \end{cases}$

$5-x$ is continuous on $[-1, 2]$ and x^2-1 is continuous on $(2, 3]$

To conclude that $g(x)$ is continuous on $[-1, 3]$, we need to examine $g(x)$ at $x=2$.

From the left on ~~x=~~ $5-x$

$$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} (5-x) = 3$$

From the right on x^2-1

$$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} (x^2-1) = 3$$

So g is continuous at $x=2$ and thus on $[-1, 3]$.

The greatest integer function

One useful function type is the step function. The greatest integer function is a type of step function.

$[\![x]\!]$ = greatest integer less than or equal to x

ex $[\![-2.1]\!] = -3$

$[\![3.5]\!] = 3$

If the domain is restricted to nonnegative numbers, this function truncates (gets rid of) the decimal portion of x.

Soln For the first 700 lbs,

$\left[\!\left[\frac{x-1}{700}\right]\!\right] = 0, \quad 0 < x \le 700$

So $C = 2750 + 290(1+0) = 3040.$

For the second 700 lbs,

$\left[\!\left[\frac{x-1}{700}\right]\!\right] = 1, \quad \cancel{or}\ 700 < x \le 1400$

So $C = 2750 + 290(1+1) = 3300$

For the third 700 lbs,

$\left[\!\left[\frac{x-1}{700}\right]\!\right] = 2, \quad 1400 < x \le 2100$

So $C = 2750 + 290(1+2) = 3620$