Section 2.4 Review

Find the value of the derivative at the given point.

3. $h(x) = x^2(3x^3 - 1)$ at $(1, 2)$

$\nwarrow x$

Approach 1: Product Rule

$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$

Let $f(x) = x^2 \quad g(x) = 3x^3 - 1$

$h'(x) = x^2(9x^2) + 2x(3x^3 - 1)$

$= 9x^4 + 6x^4 - 2x = 15x^4 - 2x$

Approach 2: Rewrite first.

$h(x) = 3x^5 - x^2$

$= 15x^4 - 2x$

At $x = 1$, $h'(1) = 15 - 2 = 13$

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11) $f(t) = \frac{2t^2 - 3}{3t + 1} \left[\frac{f(x)}{g(x)}\right]$ at $(3, \frac{3}{2})$

Quotient Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$f'(t) = \frac{(3t+1)(4t) - (2t^2 - 3)(3)}{(3t+1)^2}$

$= \frac{12t + 4t - 6t^2 + 9}{(3t+1)^2}$

$= \frac{-6t^2 + 16t + 9}{(3t+1)^2}$

$2t^2 - 3$ "$f(x)$"

$f'(x)$

$2(2t) = 4t$

$3t + 1$ "$g(x)$"

$= 3t' + 1$

$\Rightarrow 3(1t^{1-1})$ $g'(x)$

$= 3t^0$

$= 3$

$f'(3) = \frac{-6(3)^2 + 16(3) + 9}{(3 \times 3 + 1)^2} = \frac{-54 + 48 + 9}{100} = \frac{3}{100} = 0.03$

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$$P = 500\left(1 + \frac{4t}{50+t^2}\right) = \cancel{500 + 500\left(\frac{4t}{50+t^2}\right)}$$

$$\frac{dP}{dt}\left[500\left(1 + \frac{4t}{50+t^2}\right)\right]$$

$$= 500\frac{dP}{dt}\left[1 + \frac{4t}{50+t^2}\right] = 500\left[\cancelto{0}{\frac{dP}{dt}[1]} + \frac{dP}{dt}\left[\frac{4t}{50+t^2}\right]\right]$$

$$= 500\frac{dP}{dt}\left[\frac{4t}{50+t^2}\right]$$

$$= 500\left[\frac{(50+t^2)(4) - (4t)(2t)}{(50+t^2)^2}\right]$$

$$= 500\left[\frac{200 + 4t^2 - 8t^2}{(50+t^2)^2}\right]$$

$$= 500\left[\frac{200 - 4t^2}{(50+t^2)^2}\right]$$

At time $t = 2$ hours

$$P' = 500\left(\frac{200 - 4(2)^2}{(50+2^2)^2}\right)$$

$$= 500\left(\frac{200-16}{54^2}\right)$$

$$= 31.55 \text{ million microbes/hour}$$