Higher Order Derivatives

Notation:

First derivative:	y'	$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}[f(x)]$	$D_x[y]$
Second derivative:	y''	$f''(x)$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}[f(x)]$	$D_x^2[y]$
Third derivative:	y'''	$f'''(x)$	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}[f(x)]$	$D_x^3[y]$
Fourth derivative:	$y^{(4)}$	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}[f(x)]$	$D_x^4[y]$
n^{th} derivative:	$y^{(n)}$	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	$\frac{d^n}{dx^n}[f(x)]$	$D_x^n[y]$

Ex Find the first 5 derivatives of $f(x) = 2x^4 - 3x^2$

Soln $f'(x) = 8x^3 - 6x$

$f''(x) = 24x^2 - 6$

$f^{(3)}(x) = 48x$

$f^{(4)}(x) = 48$

$f^{(5)}(x) = 0.$

For an n^{th} degree polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$ the n^{th} order derivative is the constant function

$$f^{(n)}(x) = n!\, a_n$$

$$n! = n(n-1)(n-2)\cdot \dots \cdot 3\cdot 2\cdot 1$$

Ex Find $g^{(4)}(x)$ for $g(x) = \frac{1}{x}$.

Soln $g(x) = x^{-1}$

$g'(x) = -1x^{-2}$

$g''(x) = -1(-2)x^{-3} = 2x^{-3}$

$g'''(x) = 2(-3)x^{-4} = -6x^{-4}$

$g^{(4)}(x) = -6(-4)x^{-5} = 24x^{-5}$

$= \frac{24}{x^5}$

Acceleration

If $s = f(t)$ is some position function, then

$s' = f'(t)$ is the velocity and

$s'' = f''(t)$ is the acceleration.

Ex A ball is thrown upward from the top of an 80 foot cliff with an initial velocity of 64 ft/sec. Give the position function and find the acceleration at $t = 2$.

Soln $s = -16t^2 + 64t + 80$ position

$s' = -32t + 64$ velocity

$s'' = -32$ acceleration.

At $t = 2$,

velocity $= -32(2) + 64 = 0$ ft/sec

acceleration $= -32$ ft/sec^2

Ex The velocity of some car, v (ft/sec), starting from rest is

$$V = \frac{80t}{t+5}$$

Find the velocity and acceleration at $t = 60$ sec.

Soln

$$v' = \frac{(t+5)(80) - (80t)(1)}{(t+5)^2}$$

$$= \frac{400}{(t+5)^2}$$

At $t = 60$

$$V = \frac{80 \times 60}{65} = 73.8 \text{ ft/sec}$$

$$V' = \frac{400}{(65)^2} = 0.09 \text{ ft/s}^2$$