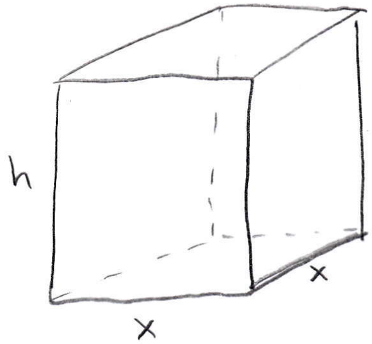
Consider an open box with a square base. The surface area is 108 in^2. What dimensions will maximize volume?



$V = hx^2$ Volume

$108 = x^2 + 4hx$ Surface area

Consider

$h = \dfrac{108 - x^2}{4x}$

So

$V = \left(\dfrac{108 - x^2}{4x}\right) x^2 = 27x - \dfrac{x^3}{4}$

Need to maximize volume over domain of x.

$0 \leq x \leq \sqrt{108}$

Using Ch 3 techniques, the maximum is at $x = 6$

which yields $h = 3$.

Ex The product of two positive numbers is 288. Minimize the sum of the second and twice the first.

Soln

1) Let x be first, y be second, S the sum.

2) $S = y + 2x$

3) Reduce: use $xy = 288 \Rightarrow y = 288x^{-1}$

So $S = 288x^{-1} + 2x$

4) Positive numbers $\rightarrow$ $x > 0$

5) $\dfrac{dS}{dx} = -288x^{-2} + 2$

$0 = 2 - 288x^2$

$x^2 = 144$

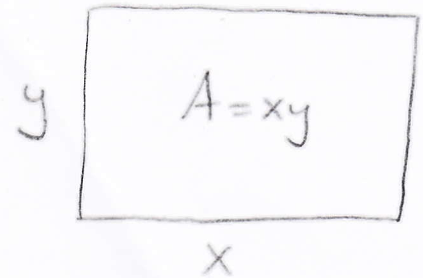
$x = \pm 12.$

Using Ch 3 techniques, find relative minimum at $x = 12$, $y = 24$.

Ex) An ecologist has 500 meters of fencing for a study plot. What should the dimensions be to maximize the enclosed area?

Soln

1) Sketch



2) $A = xy$

3) $500 = 2x + 2y$

$y = 250 - x$

So $A = x(250 - x)$

$= 250x - x^2$

4) $x, y > 0 \Rightarrow 250 - x > 0$

$x < 250$

$0 < x < 250$

5) $\frac{dA}{dx} = 250 - 2x$

$0 = 250 - 2x$

$x = 125$

$y = 250 - x$

$= 250 - 125$

$= 125$

$x = 125, \quad y = 125$