

4.3 Rules of Probability

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Goals

1. Use the addition rule to calculate the probability of one event *or* some other event occurring.
2. Use contingency tables to describe relationships between two variables.
3. Use the complement rule to calculate probabilities.

Rules of Probability

Consider a six-sided die.

$$P(\text{roll a 1 or 2}) = \frac{2 \text{ ways}}{6 \text{ outcomes}} = \frac{1}{3}$$

We get the same result by taking

$$P(\text{roll a 1}) + P(\text{roll a 2}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

It turns out this is widely applicable!

Addition Rule for Disjoint Outcomes

If A_1 and A_2 are disjoint outcomes, then the probability that one of them occurs is

$$P(A_1 \text{ or } A_2) = P(A_1) + P(A_2)$$

This can also be extended to more than two disjoint outcomes:

$$P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_k) = P(A_1) + P(A_2) + \dots + P(A_k)$$

for k disjoint outcomes.

Example: A deck of cards.

- ▶ Let A be the event that a card drawn is a diamond.
- ▶ Let B be the event it is a face card.

A : $2\blacklozenge 3\blacklozenge 4\blacklozenge 5\blacklozenge 6\blacklozenge 7\blacklozenge 8\blacklozenge 9\blacklozenge 10\blacklozenge J\blacklozenge Q\blacklozenge K\blacklozenge A\blacklozenge$

B : $J\heartsuit Q\heartsuit K\heartsuit J\clubsuit Q\clubsuit K\clubsuit J\blacklozenge Q\blacklozenge K\blacklozenge J\spadesuit Q\spadesuit K\spadesuit$

Example: A deck of cards.

The collection of cards that are diamonds or face cards (or both) is

$A \diamond 2 \diamond 3 \diamond 4 \diamond 5 \diamond 6 \diamond 7 \diamond 8 \diamond 9 \diamond 10 \diamond J \diamond Q \diamond K \diamond J \clubsuit Q \clubsuit K \clubsuit$
 $J \heartsuit Q \heartsuit K \heartsuit J \spadesuit Q \spadesuit K \spadesuit$

Looking at these cards, I can see that there are 22 of them, so

$$P(A \text{ or } B) = \frac{22}{52}$$

Example: A deck of cards.

- ▶ If I try to apply the addition rule for disjoint outcomes,

$$P(A) = \frac{13}{52}$$

and

$$P(B) = \frac{12}{52}$$

- ▶ And we get $\frac{13+12}{52} = \frac{25}{52}$.
- ▶ But we know $P(A \text{ or } B) = \frac{22}{52}$.

Example: A deck of cards.

What happened?

- ▶ When I added these, I *double counted* $J\spadesuit$, $Q\spadesuit$, and $K\spadesuit$
 - ▶ (the cards that are in both A and B).
- ▶ I need to subtract off the double count

$$\frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$

General Addition Rule

For any two events A and B , the probability that *at least* one will occur is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

- ▶ The general addition rule applies to *any* two events, even disjoint events.
 - ▶ For disjoint events, $P(A \text{ and } B) = 0$.

Inclusive or

- ▶ When we say “or”, we include the situations where:
 - ▶ A is true
 - ▶ B is true
 - ▶ both A and B are true.
- ▶ This is an *inclusive or*.

Contingency Tables

A **contingency table** is a way to summarize **bivariate data**, or data from two variables.

Smallpox in Boston (1726)

Inoculated

yes

no

total

Result

lived

238

5136

5374

died

6

Contingency Tables

- ▶ These are basically two-variable frequency distributions.
- ▶ We can convert to proportions by dividing each count by the total number of observations.

Inoculated

yes

no

total

Result

lived

0.0382

0.8252

0.8634

died

0.0010

0.1256

Inoculated

yes

no

total

Result

lived

0.0382

0.8252

0.8634

died

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Example

Inoculated

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Complements

The **complement** of an event is all of the outcomes in the sample space that are *not* in the event. For an event A , we denote its complement by A^c .

Example

- ▶ For a single roll of a six-sided die, the sample space is all possible rolls: 1, 2, 3, 4, 5, or 6.
- ▶ Let event A be rolling a 1 or a 2.
- ▶ Then A^c is rolling a 3, 4, 5, or 6.

In probability notation:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{1, 2\}$$

$$A^c = \{3, 4, 5, 6\}$$

Property

$$P(A \text{ or } A^c) = 1$$

- ▶ Using the addition rule,

$$P(A \text{ or } A^c) = P(A) + P(A^c) = 1$$

- ▶ Make sure you can convince yourself that A and A^c are *always* disjoint.

The Complement Rule

$$P(A) = 1 - P(A^c)$$

Example

Use the complement rule to find the probability a person was NOT inoculated.

Example

Consider rolling 2 six-sided dice and taking their sum.

The event of interest A is a sum less than 12. Find

1. A^c
2. $P(A^c)$
3. $P(A)$